Statistics

Lecture 9

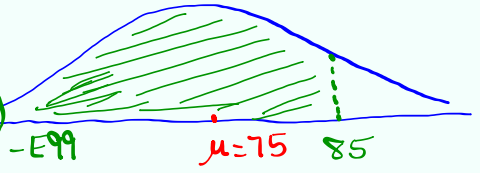
Feb 19-8:47 AM

class Qz 5:

Given $N(75, 8)$, find

1) $P(x < 85)$

= normalcdf(-E99, 85, 75, 8)

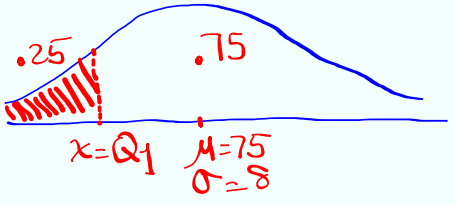


$\approx$ .894

2) $x = Q_1$, Round to a whole #.

Left Area .25
Right = .75

$x = Q_1$ = invNorm(.25, 75, 8)

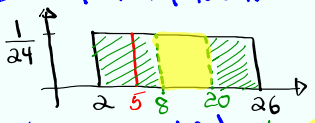


= 69.604 $\approx$ 70

Apr 24-7:49 AM

Consider a Uniform Prob. dist. for all values from 2 to 26.

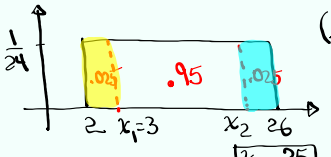
1) Draw & clearly label.



2) $P(x=5) = 0$

3) $P(x<8 \text{ or } x>20) = 1 - (20-8)\cdot\frac{1}{24} = 1 - \frac{12}{24} = \frac{12}{24} = \frac{1}{2}$

4) Find two values, rounded to whole #, that separate the middle 95% from the rest.



$x_2 = 25$

$(26-x_2)\cdot\frac{1}{24} = .025$

$26 - x_2 = 24(.025)$

$(x_1-2)\cdot\frac{1}{24} = .025$

$x_1 - 2 = 24(.025)$

$x_1 = 2 + 24(.025)$

$x_1 = 2.6$ $\quad x_1 \approx 3$

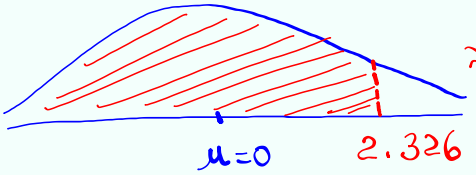
$26 - 24(.025) = x_2$

$25.4 = x_2$

$x_2 = 25$

Apr 24-8:18 AM

Find $P(Z<2.326)$ = normalcdf(-E99, 2.326, 0, 1)

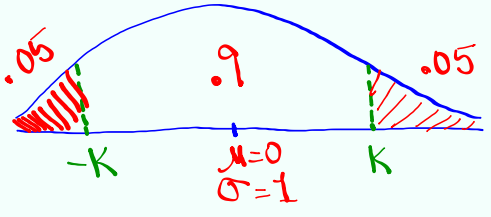


$\approx .990$

Find K such that $P(-K<Z<K) = .9$

$1 - .9 = .1$

$.1 \div 2 = .05$



$K = \text{invNorm}(.95, 0, 1) \approx 1.645$

Apr 24-8:31 AM

Credit Scores are normally dist. with the mean of 740 and standard deviation of 60. $N(740, 60)$

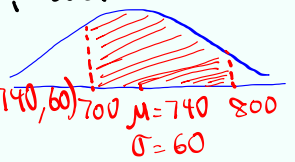
If one person is randomly selected, find the Prob. that his/her credit score

1) is between 700 & 800.

$P(700 < x < 800)$

= normalcdf(700, 800, 740, 60)

$\approx .589$

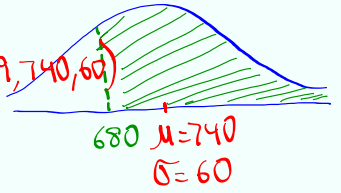


2) is greater than 680.

$P(x > 680)$

= normalcdf(680, E99, 740, 60)

$\approx .841$



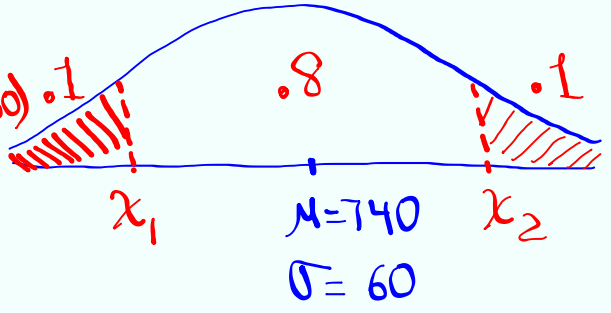
Apr 24-8:42 AM

Find two credit scores, rounded to whole #, that separate the middle 80% from the rest.

$x_1 = P_{10}$

= invNorm(.1, 740, 60)

≈ 663



$x_2 = P_{90}$

= invNorm(.9, 740, 60) ≈ 817

Apr 24-8:52 AM

Central limit theorem (CLT)

$\mu_{\bar{x}} = \mu$ $\quad \sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}$ $\quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

Suppose we take all samples of size 16 ($n = 16$)

from a pop. with mean of 150 and standard dev. of 20. Assume the pop. has a normal dist. $N(150, 20)$

$\mu_{\bar{x}} = \mu = 150$ $\quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{20}{\sqrt{16}} = \frac{20}{4} = 5$

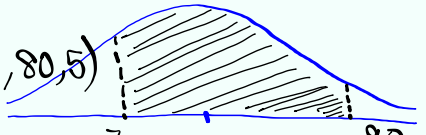
Apr 24-9:00 AM

Given $N(80, 10)$, for randomly selected groups of 4, find

$P(75 < \bar{x} < 90)$

= normalcdf(75,90,80,5)

= .819 ≈ 82%



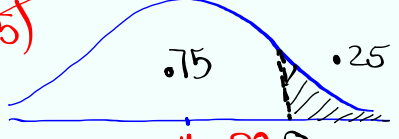
CLT $\mu_{\bar{x}} = \mu = 80$, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{4}} = 5$

$\bar{x} = Q_3$, Round to whole #

Q_3 = invNorm(.75,80,5)

= 83.372

≈ 83



Apr 24-9:05 AM

Clear all lists.

Store the Population of 2, 4, 6, 8, and 10 in L1.

Use [1-Var Stats] with L1 only for list

to find

$\mu = 6$ $\quad \sigma = 2.826$ $\quad \sigma^2 = 8$

Take all Samples of [Size 2] with replacement

From this population

2,2	2,4	2,6	2,8	2,10
4,2	4,4	4,6	4,8	4,10
6,2	6,4	6,6	6,8	6,10
8,2	8,4	8,6	8,8	8,10
10,2	10,4	10,6	10,8	10,10

Find $\bar{x}$ of each Sample

2	3	4	5	6
3	4	5	6	7
4	5	6	7	8
5	6	7	8	9
6	7	8	9	10

Apr 24-9:27 AM

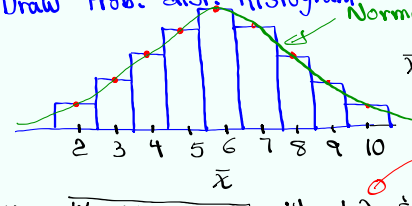
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8
5	6	7	8	9
6	7	8	9	10

25 means

$\bar{x}$	$P(\bar{x})$
2	1/25
3	2/25
4	3/25
5	4/25
6	5/25

$\bar{x}$	$P(\bar{x})$
7	4/25
8	3/25
9	2/25
10	1/25

Draw Prob. dist. histogram Normal curve



$\bar{x} \rightarrow L2$

$P(\bar{x}) \rightarrow L3$

List Freqlist

Use [1-VARSTATS] with L2 & L3.

Find

$\mu_{\bar{x}} = 6$ $\quad \sigma_{\bar{x}} = 2$ $\quad \sigma^2_{\bar{x}} = 4 = \frac{8}{2}$

$$\mu_{\bar{x}} = \mu \qquad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \qquad \sigma^2_{\bar{x}} = \frac{\sigma^2}{n}$$

CLT

Apr 24-9:35 AM

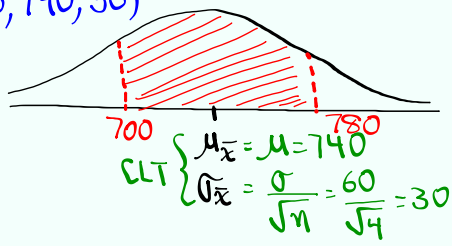
Credit Scores are normally dist. with the mean of 740 and standard deviation of 60.

If we randomly select 4 people, $n=4$, find the prob. that their mean Credit Score is between 700 & 780. $\bar{x}$

$P(700 < \bar{x} < 780)$

$= \text{normalcdf}(700, 780, 740, 30)$

$= .818$



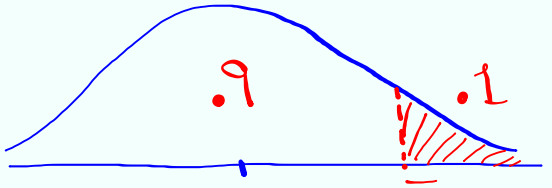
Apr 24-9:44 AM

For randomly selected 4 credit scores, $n=4$

Find the mean $\bar{x}$ that separates the top 10% from the rest. Round to whole #.

Right Area .1

Left Area .9



$\bar{x} = \text{invNorm}(.9, 740, 30)$

$= 778.447 \approx 778$

CLT $\mu_{\bar{x}} = \mu = 740$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{60}{\sqrt{4}} = 30$

Apr 24-9:50 AM

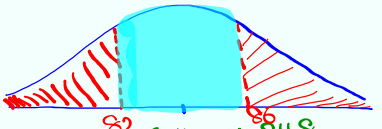
Exam Scores are normally dist. with the mean of 84.8 and standard deviation of 7.5. $N(84.8, 7.5)$

For randomly selected $n=5$ exams, find the Prob. that their mean score $\bar{x}$ is below 82 or above 86.

$P(\bar{x}<82 \text{ or } \bar{x}>86)$

$= 1 - P(82 < \bar{x} < 86)$

↑ Total Area



CLT $\mu_{\bar{x}} = \mu = 84.8$, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{7.5}{\sqrt{5}}$

$= 1 - \text{normalcdf}(82, 86, 84.8, 7.5/\sqrt{5})$

$= .562$

Apr 24-9:55 AM

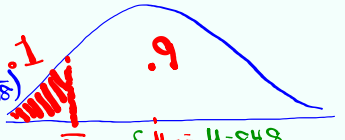
For randomly selected $n=8$ exams, find the mean $\bar{x}$, rounded to 1-decimal, that separates the bottom 10% from the rest.

$\bar{x} = P_{10}$

$= \text{invNorm}(.1, 84.8, 7.5/\sqrt{8})$

$= 81.402$

≈ 81.4



CLT $\mu_{\bar{x}} = \mu = 84.8$, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{7.5}{\sqrt{8}}$

SG 19 & 20 ✓

One page of notes both sides

Exam 2 → SG 1 – 20

Exam 2 date → May 8, 2026

Exam 2 Time → 9:30 – 11:10

Apr 24-10:05 AM

$z_{\alpha/2}$ → Area of the right-Tail

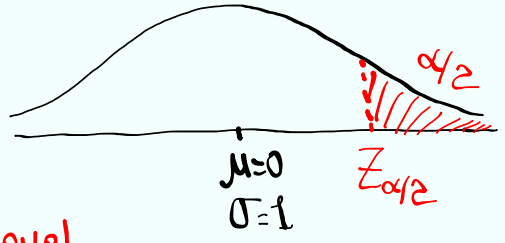
Alpha

$0 < \alpha < 1$

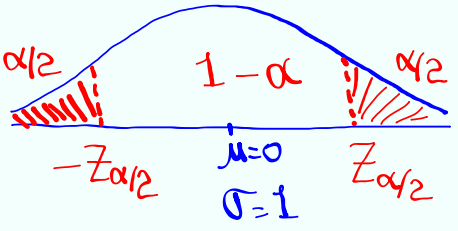
Significance level

$1-\alpha$ Confidence level

Middle Area



To find $z_{\alpha/2}$ use invNorm



Apr 24-10:18 AM

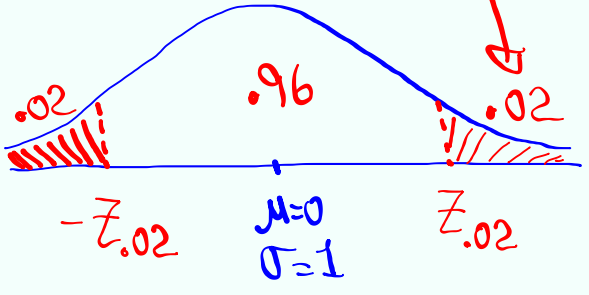
Find $z_{.02}$

$\alpha/2 = .02$

$\alpha = .04$

$1-\alpha = .96$

Middle Area



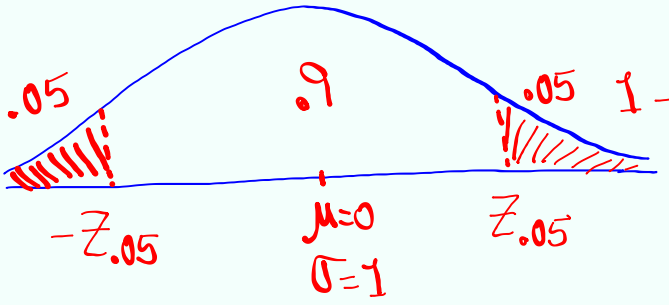
$z_{.02} = \text{invNorm}(.98, 0, 1) \approx 2.054$

Apr 24-10:23 AM

Find $z_{\alpha/2}$ for 90% conf. level.

Middle Area

$1-\alpha$



$1-\alpha = .9$

$\alpha = .1$

$\alpha/2 = .05$

$z_{.05} = \text{invNorm}(.95, 0, 1) \approx \boxed{1.645}$

Apr 24-10:26 AM

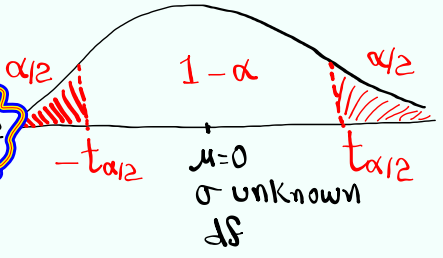
t – Distribution

1) Graph is symmetric, bell-shape with total area 1.

2) $\mu = 0$ but σ is unknown

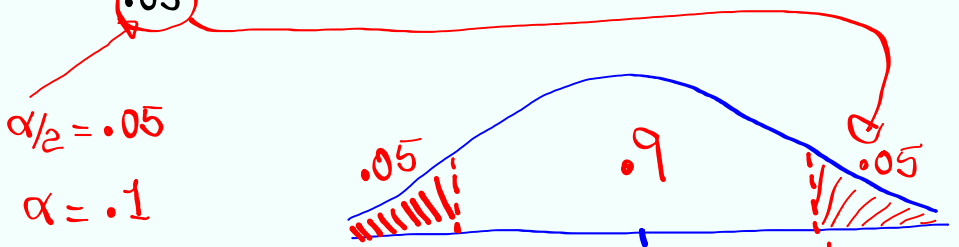
3) It comes with degrees of freedom **df**.
df will be given or determined by topic.

How to find $t_{\alpha/2}$
invt



Apr 24-10:30 AM

Find $t_{.05}$ with $df = 9$



$\alpha/2 = .05$

$\alpha = .1$

$1 - \alpha = .9$

Middle Area

Conf. level: .9

σ unknown

$df = 9$

$t_{.05} = invT(.95, 9) = \boxed{1.833}$

Apr 24-10:35 AM

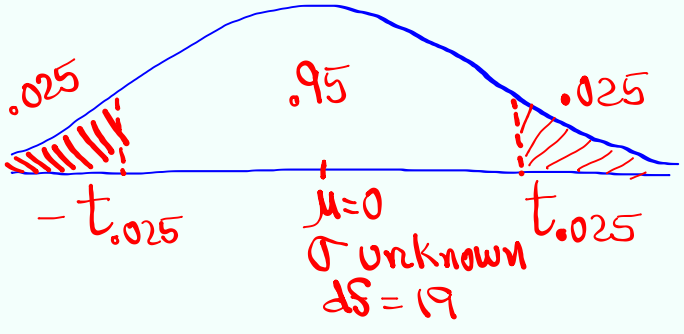
Find $t_{\alpha/2}$ for 95% Confidence level with $df = 19$.

Middle Area = .95

$1 - \alpha = .95$

$\alpha = .05$

$\alpha/2 = .025$



σ unknown

$df = 19$

$t_{.025} = invT(.975, 19) \approx \boxed{2.093}$

Apr 24-10:39 AM

What is degrees of freedom?

For now it is given to us.

Later, it will be determined by topics.

Here is an example:

Pick 3 numbers that have a Sum of 10

Tiffany 4

Nemo 4 **No freedom**

Me 2

df = 2

There are 20 students, I bring 20 donuts

You can pick 1 donut.

Ivan 20 choices

Ethan 19 "

Faith 18 "

⋮

Nemo 1 (No choices)

19 had choices

df = 19

Apr 24-10:43 AM

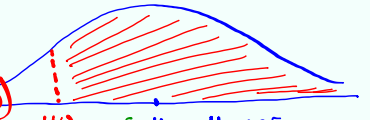
Class QZ 6 open notes:

Given $N(125, 15)$, $n = 4$

1) Find $P(\bar{x} > 110)$

= normalcdf(110, E99, 125, 7.5)

$\approx$ .977



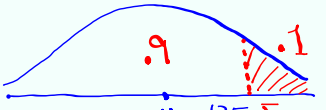
CLT $\mu_{\bar{x}} = \mu = 125$, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{15}{\sqrt{4}} = 7.5$

2) Find $\bar{x} = P_{90}$, Round to whole #.

$\bar{x}$ = invNorm(.9, 125, 7.5)

= 134.612

$\approx$ 135



CLT $\mu_{\bar{x}} = 125$ $\bar{x}$, $\sigma_{\bar{x}} = 7.5$

Apr 24-10:50 AM